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A Multiaxial Strength Criterion for Composites

P. Y. Tang

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This document is the preprint for a paper to be published in the Proceedings of the First Navy Independent Research/Independent Exploratory Development (IR/IED) Symposium that was held at Johns Hopkins University, 20 to 22 June 1988. The document presents the development of a piecewise quadratic strength tensor theory for composites with orthotropic, transversely isotropic, and isotropic material symmetries. The proposed strength criterion improves the best available quadratic failure theory for such composites, the Tsai and Wu quadratic strength tensor theory, by including stress terms that can reflect different failure mechanisms of the composites under tension and compression. To demonstrate the applicability of the proposed theory to composites, extensive and good correlations are shown between the theory and the biaxial fracture data of five composite material systems: graphite/epoxy, graphite particulate, graphite/aluminum, glass/epoxy, and organic textolite composites.

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A MULTIAXIAL STRENGTH CRITERION FOR COMPOSITES*

ABSTRACT

This paper presents the development of a piecewise quadratic strength tensor theory for composites with orthotropic, transversely isotropic, and isotropic material symmetries. The proposed strength criterion improves the best available quadratic failure theory for such composites, the Tsai and Wu quadratic strength tensor theory, by including stress terms that can reflect different failure mechanisms of the composites under tension and compression. To demonstrate the applicability of the proposed theory to composites, extensive and good correlations are shown between the theory and the biaxial fracture data of five composite material systems: graphite epoxy, graphite particulate, graphite aluminum, glass epoxy, and organic textolite composites.

INTRODUCTION

A multiaxial strength (failure) criterion is an equation to be satisfied by the stress components under which failure occurs. In general, six stress components are used to define the stress state and a strength criterion can be geometrically viewed as a failure surface in the six-dimensional stress space. The failure surface has to be closed[1,2] to ensure that the material strength is finite in all directions. In addition, a strength criterion for composites is required at least to account for the following general material characteristics: (i) volume compressibility, (ii) differing tension and compression strengths, and (iii) orthotropic, transversely isotropic, and isotropic material symmetries for orthotropic, transversely isotropic, and quasi-isotropic composites, respectively.

As is well-known, the Tsai-Wu quadratic strength tensor theory[1] satisfies all of the above requirements and encompasses all other quadratic strength (failure) criteria used for composites. For a general anisotropic solid, this theory can be written as**

$$f(\sigma_k) = F_i\,\sigma_i + F_{ij}\,\sigma_i\,\sigma_j = 1\ ,\ (i,j,k = 1,\dots,6)\ , \tag{1}$$

where f is a scalar function, σ_k is the contracted notation of the second rank stress tensor⁺, and F_i and F_{ij} are the strength tensors of rank two and four, respectively. Without loss of generality, it is assumed that

$$F_{ij} = F_{ji}\ . \tag{2}$$

In addition, constraints (usually referred to as the stability conditions) must be imposed on the strength tensor F_{ij} to ensure that the material strength is finite in all directions. More specifically, F_{ij} must be positive definite:

$$F_{ij}\,\sigma_i\,\sigma_j > 0\ , \tag{3}$$

at all points σ_i in the six-dimensional stress space. Geometrically, Eq. (3) is a necessary and sufficient condition to ensure that the failure surface represented by the quadratic polynomial of Eq. (1) is closed and ellipsoidal.

In the biaxial stress plane, the Tsai-Wu criterion represents a single ellipse. In general, a single continuous ellipse cannot satisfactorily represent the biaxial data of composites in all four stress quadrants. To account for the nonelliptical characteristics of the biaxial fracture data of composites, Chamis[3] and Rosen[4] suggested to use the Tsai-Wu quadratic criterion with different F_{ij} ($i \neq j$) for different stress quadrants. Beyond having more coefficients for better data fit, there is no physical or mathematical justification.[5] Another approach to improve the Tsai-Wu quadratic criterion for composites application was to include the cubic terms in Eq. (1).[6,7,8] Here, enormous numbers of sixth order strength tensor components are involved that have to be reduced by ad hoc assumptions. Moreover, having cubic stress terms, the failure surface becomes open-ended.[1]

Without suffering any of these shortcomings, Tang and Kuei[9] improved Tsai and Wu's theory in correlating the biaxial strength data of (monotonous) polycrystalline graphite, which show similar nonelliptical characteristics to composite data. Recognizing the fact that such characteristics may be due to different fracture mechanisms being operative under different states of biaxial stresses with different combinations of tensile and compressive stresses,[10] they added to the Tsai-Wu criterion the quadratic stress terms with the absolute value of the linear combination of stress components. The resulting piecewise quadratic strength tensor theory can be written as

*The author gratefully acknowledges the financial support from Space and Naval Warfare Systems Command under Independent Research Program. He is also thankful to Mr. R.P. Johnson for his assistance in preparing the curve-fitting and plotting results presented in this paper.

**Unless otherwise indicated, the usual summation convention over a repeated index is used throughout this paper.

⁺With reference to a rectangular Cartesian coordinate system (i.e., xyz or equivalently, $x_1x_2x_3$ system): $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$, $\sigma_3 = \sigma_z$, $\sigma_4 = \sigma_{xy}$, $\sigma_5 = \sigma_{yz}$, $\sigma_6 = \sigma_{zx}$.

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$$f(\sigma_k) = F_i\,\sigma_i + F_{ij}\,\sigma_i\,\sigma_j + H_i\,\sigma_i\,|H_j\,\sigma_j| = 1\ ,\ (i,j,k = 1,\ldots,6)\ , \tag{4}$$

where H_i is a second rank tensor.

The above equation holds for a general anisotropic material. Tang and Kuei then reduced all the results pertaining to the anisotropic material to a transversely isotropic and an isotropic graphite and demonstrated good correlations between the theory and the biaxial fracture data of graphite.

In view of such good correlations with biaxial graphite data which show similar nonelliptical characteristics to composite data, it was proposed that the piecewise quadratic strength tensor theory be developed for composites.[11]

PIECEWISE QUADRATIC STRENGTH TENSOR THEORY

Presented below are the general results pertaining to the proposed piecewise quadratic strength tensor theory for composites, including general anisotropic, orthotropic, transversely isotropic, and isotropic materials. The results contain the explicit expressions for the strength criteria of materials with various material symmetries, the restrictions imposed on the components of the strength tensors occurring in these criteria, and the geometric meaning of these criteria.

ANISOTROPIC MATERIAL

The proposed multiaxial strength criterion for a general anisotropic material was given in Eq. (4) with F_{ij} being assumed symmetric as shown by Eq. (2). With this assumption, there are 6 independent strength tensor components for each of F_i and H_i, and 21 for F_{ij}. These numbers for a general anisotropic material can usually be reduced substantially for a material having a certain material symmetry. Such reductions will be shown in the next three subsections.

Equation (4) can be decomposed into two equations:

$$F_i\,\sigma_i + (F_{ij} + H_i\,H_j)\,\sigma_i\,\sigma_j = 1\ , \tag{5}$$

for all σ_i with

$$H_i\,\sigma_i \geq 0\ , \tag{6}$$

and

$$F_i\,\sigma_i + (F_{ij} - H_i\,H_j)\,\sigma_i\,\sigma_j = 1\ , \tag{7}$$

for all σ_i with

$$H_i\,\sigma_i < 0\ . \tag{8}$$

The stability conditions to ensure the closure of each of the failure surfaces represented by Eqs. (5) and (7), respectively, are

$$(F_{ij} + H_i\,H_j)\,\sigma_i\,\sigma_j > 0\ , \tag{9}$$

for all σ_i satisfying Eq. (6), and

$$(F_{ij} - H_i\,H_j)\,\sigma_i\,\sigma_j > 0\ , \tag{10}$$

for all σ_i satisfying Eq. (8). Clearly, Eqs. (9) and (10) are the stability conditions required to ensure the closure of the entire piecewise failure surface represented by Eq. (4).

Geometrically, the failure surfaces represented by Eqs. (5) and (7), respectively, with the restrictions made by Eqs. (9) and (10) on the strength tensors F_{ij} and H_i are two ellipsoids in the two half spaces defined by Eqs. (6) and (8). Thus, the failure surface represented by Eq. (4) with the strength tensors F_{ij} and H_i satisfying the stability conditions given by Eqs. (9) and (10) is a piecewise ellipsoid in the six-dimensional stress space. Hence, the proposed quadratic strength tensor theory has been referred to as the piecewise quadratic strength tensor theory.

ORTHOTROPIC MATERIAL

For an orthotropic material with the reference coordinate planes coinciding with the planes of material symmetry, based on the invariance requirements[12] of orthotropy material symmetry, the stress dependent function f in Eq. (4) must be expressible as a polynomial in the seven quantities: σ_1, σ_2, σ_3, σ_4^2, σ_5^2, σ_6^2, $\sigma_4\sigma_5\sigma_6$ (or alternatively, I_3), where I_3 is a stress invariant given by

$$I_3 = \sigma_1^{\,3} + \sigma_2^{\,3} + \sigma_3^{\,3} + 3\sigma_1\,(\sigma_4^{\,2} + \sigma_6^{\,2}) + 3\sigma_2\,(\sigma_4^{\,2} + \sigma_5^{\,2}) + 3\sigma_3\,(\sigma_5^{\,2} + \sigma_6^{\,2}) + 6\,\sigma_4\,\sigma_5\,\sigma_6\ . \tag{11}$$

Hence, the explicit expression for the quadratic function f defined by Eq. (4) can be given in terms of the above-mentioned quantities as

$$f = F_1 \sigma_1 + F_2 \sigma_2 + F_3 \sigma_3 + F_{11} \sigma_1^2 + 2F_{12} \sigma_1 \sigma_2 + 2F_{13} \sigma_1 \sigma_3 + F_{22} \sigma_2^2 + 2F_{23} \sigma_2 \sigma_3 + F_{33} \sigma_3^2$$
$$+ F_{44} \sigma_4^2 + F_{55} \sigma_5^2 + F_{66} \sigma_6^2 + (H_1 \sigma_1 + H_2 \sigma_2 + H_3 \sigma_3)|H_1 \sigma_1 + H_2 \sigma_2 + H_3 \sigma_3| = 1 . \quad (12)$$

Comparing Eq. (12) with Eq. (4), it can be seen that

$$F_4 = F_5 = F_6 = 0 , \; H_4 = H_5 = H_6 = 0 ,$$
$$F_{14} = F_{15} = F_{16} = F_{24} = F_{25} = F_{26} = F_{34} = F_{35} = F_{36} = F_{45} = F_{46} = F_{56} = 0 . \quad (13)$$

With the above results, it is clear that there are only three independent strength tensor components for each of F_i and H_i (i.e., (F_1, F_2, F_3) and (H_1, H_2, H_3)), and only nine for F_{ij} (i.e., F_{11}, F_{22}, F_{33}, F_{44}, F_{55}, F_{66}, F_{12}, F_{23}, F_{13}). As noted earlier, the numbers of independent strength tensor components for a general anisotropic material have been substantially reduced due to orthotropy material symmetry.

The strength constants mentioned above are not free material parameters because they are restricted by the stability conditions Eqs. (9) and (10). For an orthotropic material, using Eq. (13), the independent restrictions on F_{ij} and H_i can be obtained as

$$F_{11} \pm H_1^2 > 0 , \; F_{22} \pm H_2^2 > 0 , \; F_{33} \pm H_3^2 > 0 , \; F_{44} > 0 , \; F_{55} > 0 , \; F_{66} > 0 ,$$
$$(F_{11} \pm H_1^2)(F_{22} \pm H_2^2) - (F_{12} \pm H_1 H_2)^2 > 0 , \; (F_{22} \pm H_2^2)(F_{33} \pm H_3^2) - (F_{23} \pm H_2 H_3)^2 > 0 ,$$
$$(F_{33} \pm H_3^2)(F_{11} \pm H_1^2) - (F_{13} \pm H_1 H_3)^2 > 0 ,$$
$$(F_{11} \pm H_1^2)(F_{22} \pm H_2^2)(F_{33} \pm H_3^2) + 2(F_{12} \pm H_1 H_2)(F_{23} \pm H_2 H_3)(F_{13} \pm H_1 H_3)$$
$$- (F_{11} \pm H_1^2)(F_{23} \pm H_2 H_3)^2 - (F_{22} \pm H_2^2)(F_{13} \pm H_1 H_3)^2 - (F_{33} \pm H_3^2)(F_{12} \pm H_1 H_2)^2 > 0 . \quad (14)$$

TRANSVERSELY ISOTROPIC MATERIAL

For a transversely isotropic material with the x_3 axis being parallel to the axis of rotational symmetry, based on the invariance requirements[12] of the transverse isotropy material symmetry, the function f in Eq. (4) must be expressible as a polynomial in the five quantities I_1, I_2, I_3, σ_3, and $\sigma_5^2 + \sigma_6^2$, where I_1 and I_2 are stress invariants given by

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 , \; I_2 = (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) + 2(\sigma_4^2 + \sigma_5^2 + \sigma_6^2) , \quad (15)$$

and I_3 is the stress invariant given by Eq. (11). Thus, the quadratic function f defined by Eq. (4) can be expressed as

$$f = a_0 I_1 + a_1 \sigma_3 + [b_0 I_1^2 + b_1 I_1 \sigma_3 + b_2 \sigma_3^2 + b_3 I_2 + b_4(\sigma_5^2 + \sigma_6^2)] + (c_0 I_1 + c_1 \sigma_3)|c_0 I_1 + c_1 \sigma_3| = 1 . \quad (16)$$

A comparison of Eq. (16) with Eq. (12) using Eqs. (15) and (11) leads to the following results for the nonvanishing strength tensor components.

$$F_1 = F_2 = a_0 , \; F_3 = a_0 + a_1 , \; H_1 = H_2 = c_0 , \; H_3 = c_0 + c_1 , \; F_{11} = F_{22} = b_0 + b_3 , \; F_{33} = b_0 + b_1 + b_2 + b_3 ,$$
$$F_{44} = 2b_3 = 2(F_{11} - F_{12}) , \; F_{55} = F_{66} = 2b_3 + b_4 , \; F_{12} = b_0 , \; F_{23} = F_{13} = b_0 + \frac{b_1}{2} . \quad (17)$$

Substituting these results into Eq. (12), the expression for f in Eq. (4) becomes

$$f = F_1(\sigma_1 + \sigma_2) + F_3 \sigma_3 + F_{11}(\sigma_1^2 + \sigma_2^2 + 2\sigma_4^2) + F_{33} \sigma_3^2 + F_{55}(\sigma_5^2 + \sigma_6^2) + 2F_{12}(\sigma_1 \sigma_2 - \sigma_4^2)$$
$$+ 2F_{13}(\sigma_1 + \sigma_2)\sigma_3 + [H_1(\sigma_1 + \sigma_2) + H_3 \sigma_3]|H_1(\sigma_1 + \sigma_2) + H_3 \sigma_3| = 1 . \quad (18)$$

From Eq. (17) or Eq. (18), it is clear that there are only two independent strength tensor components for each of F_i and H_i (i.e., (F_1, F_3) and (H_1, H_3)) and only five for F_{ij} (i.e., F_{11}, F_{33}, F_{55}, F_{12}, F_{13}). The independent stability restrictions on these strength constants can be obtained by substituting Eq. (17) into Eq. (14) as

$$F_{11} + F_{12} \pm 2H_1^2 > 0 , \; F_{11} - F_{12} > 0 , \; F_{33} \pm H_3^2 > 0 ,$$
$$(F_{33} \pm H_3^2)(F_{11} + F_{12} \pm 2H_1^2) - 2(F_{13} \pm H_1 H_3)^2 > 0 , \; F_{55} > 0 . \quad (19)$$

ISOTROPIC MATERIAL

For an isotropic material, based on the invariance requirements[12] of the isotropy material symmetry, the stress dependent function f defined in Eq. (4) must be expressible as a polynomial in the stress invariants I_1, I_2, and I_3 defined in Eqs. (15) and (11). In light of this and following the similar derivations of the last subsection, the expression for f defined by Eq. (4) can be obtained as

$$f = F_1 I_1 + F_{11} I_2 + F_{12}(I_1^2 - I_2) + H_1 I_1 |H_1 I_1| = 1 , \tag{20}$$

with

$$F_1 = F_2 = F_3 , \; H_1 = H_2 = H_3 , \; F_{11} = F_{22} = F_{33} , \; F_{44} = F_{55} = F_{66} = 2(F_{11} - F_{12}) , \; F_{12} = F_{23} = F_{13} . \tag{21}$$

Furthermore, the independent stability conditions on the four independent strength tensor components (i.e. F_1, H_1, F_{11}, and F_{12}) for an isotropic material are

$$F_{11} + F_{12} \pm 2H_1^2 > 0 , \; F_{11} - F_{12} > 0 , \; (F_{11} \pm H_1^2)(F_{11} + F_{12} \pm 2H_1^2) - 2(F_{12} \pm H_1^2)^2 > 0 . \tag{22}$$

GENERAL REMARKS

Along with the developments made so far, the following should be clear:

(1) The numbers of independent strength tensor components for anisotropic, orthotropic, transversely isotropic, and isotropic materials, respectively, are 33, 15, 9, and 4 in the proposed theory and 27, 12, 7, and 3 in the Tsai-Wu theory.

(2) When H_i (i = 1,...,6) are all vanishing, the proposed theory degenerates into the Tsai-Wu theory and various results obtained in this section reduce to the corresponding results for Tsai and Wu's theory.

(3) In the six-dimensional stress space, the proposed criterion represents a piecewise ellipsoid made of two ellipsoids in two half spaces, whereas the Tsai-Wu criterion represents a single ellipsoid.

PROPOSED BIAXIAL STRENGTH CRITERION

To facilitate the correlations of the proposed theory with the biaxial fracture data on composites, results obtained in the last section for a general multiaxial stress state are reduced in this section for a biaxial stress state:

$$\sigma_1 \neq 0 , \; \sigma_3 \neq 0 , \; \sigma_2 = \sigma_4 = \sigma_5 = \sigma_6 = 0 \tag{23}$$

ANISOTROPIC MATERIAL

Substituting Eq. (23) into Eq. (4), we obtain

$$f = F_1 \sigma_1 + F_3 \sigma_3 + F_{11} \sigma_1^2 + F_{33} \sigma_3^2 + 2F_{13} \sigma_1 \sigma_3 + (H_1 \sigma_1 + H_3 \sigma_3)|H_1 \sigma_1 + H_3 \sigma_3| = 1 , \tag{24}$$

which is the desired biaxial strength (failure) criterion for an anisotropic material.

Using Eq. (24), the stability conditions on the seven strength constants (F_1, F_3; F_{11}, F_{33}, F_{13}; H_1, H_3) appearing in Eq. (24) can be obtained as*

$$F_{11} \pm H_1^2 > 0 , \; (F_{11} \pm H_1^2)(F_{33} \pm H_3^2) - (F_{13} \pm H_1 H_3)^2 > 0 , \tag{25}$$

which are a subset of the stability conditions given by Eqs. (9) and (10) for a general multiaxial stress state.

Geometrically, Eq. (24) with the strength constants satisfying the stability conditions given by Eq. (25) represents a piecewise ellipse in the biaxial stress plane. This piecewise ellipse is made of a single ellipse represented by

$$f = F_1 \sigma_1 + F_3 \sigma_3 + (F_{11} + H_1^2) \sigma_1^2 + (F_{33} + H_3^2) \sigma_3^2 + 2(F_{13} + H_1 H_3) \sigma_1 \sigma_3 = 1 , \tag{26}$$

in the half plane

$$H_1 \sigma_1 + H_3 \sigma_3 \geq 0 , \tag{27}$$

*Alternatively, Eq. $(25)_1$ can be replaced by $F_{33} \pm H_3^2 > 0$.

and another single ellipse represented by

$$f = F_1 \sigma_1 + F_3 \sigma_3 + (F_{11} - H_1^2) \sigma_1^2 + (F_{33} - H_3^2) \sigma_3^2 + 2(F_{13} - H_1 H_3) \sigma_1 \sigma_3 = 1 , \tag{28}$$

in the half plane

$$H_1 \sigma_1 + H_3 \sigma_3 < 0 . \tag{29}$$

ORTHOTROPIC AND TRANSVERSELY ISOTROPIC MATERIALS

The biaxial strength criteria for an orthotropic material and a transversely isotropic material can be obtained by substituting Eq. (23) into Eqs. (12) and (18), respectively. They are found to be identical to Eq. (24) for an anisotropic material. Consequently, the stability conditions on the seven strength constants (F_1, F_3; F_{11}, F_{33}, F_{13}, H_1, H_3) for an orthotropic or a transversely isotropic material are also identical to Eq. (25) for an anisotropic material.

ISOTROPIC MATERIAL

Substitution of the biaxial stress condition given by Eq. (23) into Eq. (20) and use of Eqs. (11) and (15) lead to the biaxial strength criterion for an isotropic material as

$$f = F_1 (\sigma_1 + \sigma_3) + F_{11} (\sigma_1^2 + \sigma_3^2) + 2F_{12} \sigma_1 \sigma_3 + H_1 (\sigma_1 + \sigma_3) | H_1 (\sigma_1 + \sigma_3) | = 1 . \tag{30}$$

Using Eq. (30), the stability conditions on the four strength constants (F_1; F_{11}, F_{12}, H_1) appearing in Eq. (30) can be obtained as

$$F_{11} + F_{12} \pm 2H_1^2 > 0 , \; F_{11} - F_{12} > 0 , \tag{31}$$

which constitute only a subset of the stability conditions given by Eq. (22) for a general multiaxial stress state.

REMARKS ON COMPARISON WITH TSAI-WU'S BIAXIAL CRITERION

To facilitate the comparison between the proposed and the Tsai-Wu biaxial criterion in correlating the biaxial fracture data of composites, the following remarks are collected here:

(1) As compared with the Tsai-Wu biaxial criterion, the proposed biaxial criterion contains only 2, 2, and 1 additional strength constants for orthotropic, transversely isotropic, and isotropic materials, respectively.

(2) As remarked earlier, when all the components of the strength tensor H_i are set vanishing, all the results obtained in this section degenerate into those pertaining to the Tsai-Wu theory.

(3) In the biaxial stress plane, the proposed biaxial criterion represents a piecewise ellipse consisting of two ellipses in two half planes, whereas the Tsai-Wu's biaxial criterion represents a single ellipse.

CORRELATIONS WITH BIAXIAL FRACTURE DATA OF COMPOSITES

To demonstrate the applicability of the proposed piecewise quadratic strength tensor theory to composites, comparisons are made in this section between the proposed theory and the available biaxial fracture data on composites. These data cover a wide spectrum of composite material systems: graphite/epoxy,[13] graphite particulate,[10] graphite/aluminum,[14] glass/epoxy,[15,16,17] and organic fiber-reinforced textolite.[18] They were all obtained from tubular specimens subjected simultaneously to an axial load and internal and/or external fluid pressure, except those on graphite/aluminum unidirectional composite which were obtained from flat cruciform specimens under biaxial in-plane loadings. In correlating the data for tube specimens, the circumferential (i.e., tangential) direction will be designated as the direction for x (i.e., x_1) axis and the direction along the axis of the tube as the direction for z (i.e., x_3) axis. For the flat cruciform specimens, the fiber and its perpendicular directions will be identified as the directions for x and z axes, respectively.

Also, to show the improvements of the proposed theory over the Tsai-Wu theory, comparisons are made between the two theories. In these comparisons, Eq. (30) of the proposed theory and its degenerated version for the Tsai-Wu theory are used for $[0/90/0/90]_s$ graphite/epoxy laminate which is treated as an isotropic material, while Eq. (24) of the proposed theory and its degenerated version for the Tsai-Wu theory are used for other composites which are either transversely isotropic (graphite particulate) or orthotropic (the rest of composites).

Tables I and II, respectively, present the strength constants* for the five composite systems, least-square-fitted by the above-mentioned equations pertaining to the Tsai-Wu theory and the proposed theory, without violating the appropriate stability restrictions on the fitted strength constants.

*In presenting the strength constants for the quasi-isotropic graphite/epoxy laminate, the results, $F_{12} = F_{13}$, of Eq. (21)[10, 11] have been used.

Figures 1 to 11 present the correlations of the proposed theory and the Tsai-Wu theory with the biaxial fracture data of various composites:

(1) 0-deg graphite/epoxy lamina[13] (Fig. 1) ;

(2) $[0/90/0/90]_s$ graphite/epoxy laminate[13] (Fig. 2) ;

(3) JT-50 graphite-based refractory particulate composite with the longitudinal axis of the test specimens being parallel to the symmetry axis of the material[10] (Fig. 3) ;

(4) Unidirectional graphite/aluminum composite[14] (Fig. 4) ;

(5) Satin- and linen-weave glass-reinforced plastics with the fill direction of the material coincident with the direction of the tubular test piece axis[15] (Figs. 5, 6) ;

(6) Circumferentially wound unidirectional glass/epoxy laminate[16] (Fig. 7) ;

(7) $[90/\pm 30/90]$ glass fiber reinforced laminate[16] (Fig. 8) ;

(8) Cross-ply glass/epoxy laminate with the first and the third layers being oriented along the circumference and the second layer along the tube axis[17] (Fig. 9) ;

(9) Helically wound glass/epoxy laminate with the fiber reinforcement orientation at an angle of $\pm 45°$ to the tube axis[17] (Fig. 10) ;

(10) Organic textolite with the fill direction of the reinforcing fabric coincident with the tube axis direction[18] (Fig. 11).

For the data presented in Figs. 1, 5, 6, 7, 10, and 11, which possess elliptical characteristics, both theories correlate equally well. For the data presented in Fig. 2, the proposed theory correlates better than the Tsai-Wu theory and predicts significantly different results, at least in the third (i.e., compression-compression) stress quadrant, from the Tsai-Wu theory. For the data presented in Figs. 3, 4, 8, and 9, which possess nonelliptical chracteristics, the proposed theory shows significant improvements over the Tsai-Wu theory in the correlations.

CONCLUSIONS

Good correlations between the theory and the biaxial fracture data have been demonstrated for all five composite material systems: graphite/epoxy, graphite particulate, graphite/aluminum, glass/epoxy, and organic textolite. Furthermore, significant improvements of the proposed theory over the Tsai-Wu theory have been shown for the cases where the biaxial data have nonelliptical characteristics. From these results, the following conclusions are reached:

(1) The proposed piecewise quadratic strength tensor theory is applicable to the composites.

(2) The proposed theory can significantly improve Tsai-Wu's quadratic strength tensor theory for composite applications.

NOMENCLATURE

f	=	Strength Function.
F_i, F_{ij}, H_i, $(i, j = 1,...,6)$	=	Strength Tensors.
I_i, $(i = 1, 2, 3)$	=	Stress Invariants.
x_i, $(i = 1, 2, 3)$	=	Rectangular Cartesian Coordinates.
σ_i, $(i = 1,...,6)$	=	Stress Tensor.
σ_x, σ_y, σ_z	=	Normal Stress Components.
σ_{xy}, σ_{yz}, σ_{zx}	=	Shear Stress Components.

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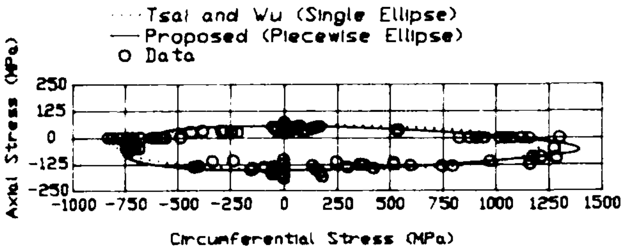
Table I Fitted Strength Constants of the Tsai-Wu Criterion.

COMPOSITE \ CONSTANT	F_1 10^{-3} (MPa*)$^{-1}$	F_3 10^{-3} (MPa)$^{-1}$	F_{11} 10^{-5} (MPa)$^{-2}$	F_{33} 10^{-5} (MPa)$^{-2}$	F_{13} 10^{-5} (MPa)$^{-2}$
Graphite/Epoxy Lamina	0.4877	10.0958	0.1292	11.0444	0.0386
[0/90/0/90]s Graphite/Epoxy Laminate	1.0332	1.0332	0.5795	0.5795	0.4154
H-50 Graphite Particulate	6.0624	4.9071	5.9825	5.3231	2.0747
Graphite/Aluminum Lamina	0.1357	4.3759	0.3478	2.9786	0.0137
Satin-Weave Glass/Epoxy	1.1370	0.7599	0.8483	1.6218	0.0472
Linen-Weave Glass/Epoxy	1.9144	1.0819	1.2304	2.2617	0.1358
Unidirectional Glass/Epoxy Laminate	0.3311	15.2337	0.1076	16.7720	0.4108
[90/±30/90] Glass/Epoxy Laminate	2.0058	0.8168	2.0058	0.5241	0.5283
Cross-ply Glass/Epoxy Laminate	1.7605	2.2612	0.4670	1.0530	0.1013
Helically Wound Glass/Epoxy Laminate	1.2840	0.3224	4.3373	4.7796	4.2193
Organic Textile	2.5515	3.1901	1.3469	0.6906	0.2332

Table II Fitted Strength Constants of the Proposed Theory

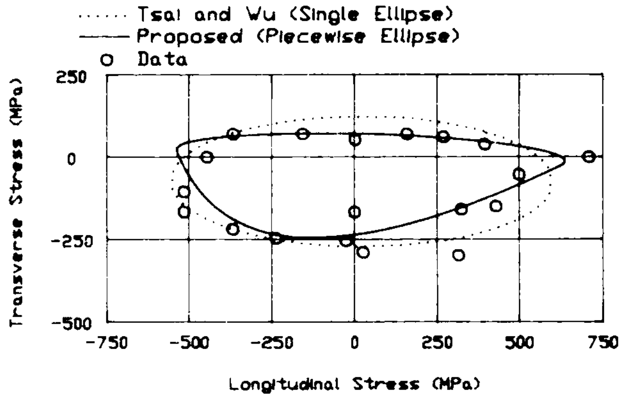
COMPOSITE \ CONSTANT	F_1 10^{-3} (MPa*)$^{-1}$	F_3 10^{-3} (MPa)$^{-1}$	F_{11} 10^{-5} (MPa)$^{-2}$	F_{33} 10^{-5} (MPa)$^{-2}$	F_{13} 10^{-5} (MPa)$^{-2}$	H_1 10^{-3} (MPa)$^{-1}$	H_3 10^{-3} (MPa)$^{-1}$
Graphite/Epoxy Lamina	0.9756	0.9734	0.1280	17.9832	0.4822	0.7148	11.4688
[0/90/0/90]s Graphite/Epoxy Laminate	3.0015	3.0015	0.6029	0.6029	0.2551	1.9927	1.9927
H-50 Graphite Particulate	11.0493	5.9855	7.7056	61.6362	12.2767	4.8345	24.8041
Graphite/Aluminum Lamina	0.4212	0.8186	0.3012	12.6053	0.3191	0.4853	11.1480
Satin-Weave Glass/Epoxy	0.4849	0.0337	0.9957	1.6064	0.1791	2.3382	1.4630
Linen-Weave Glass/Epoxy	0.2138	0.0016	1.4293	2.2845	0.4558	2.5631	2.0856
Unidirectional Glass/Epoxy Laminate	0.7923	21.2772	0.1154	15.1689	0.5245	0.5136	7.5298
[90/±30/90] Glass/Epoxy Laminate	2.7866	0.7685	6.1122	0.4205	0.4778	7.5588	0.1482
Cross-ply Glass/Epoxy Laminate	1.1279	2.9361	2.2386	0.8883	0.3208	4.6894	0.8264
Helically Wound Glass/Epoxy Laminate	1.2840	0.3224	4.3373	4.7796	4.2193	0.9807	1.3922
Organic Textile	2.0000	3.1829	1.4021	0.6739	0.2452	1.4209	0.2329

*Note: 1 MPa = 0.145 Ksi



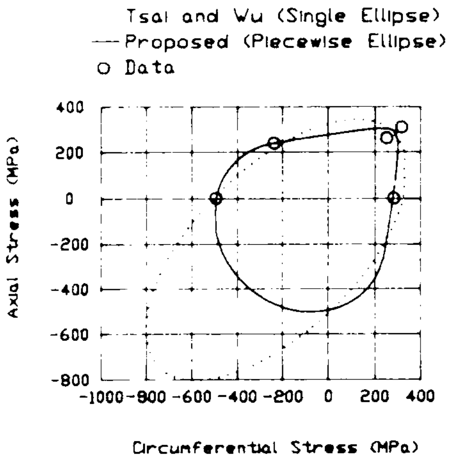
(Note 1 MPa = 0.145 Ksi)

Fig. 1 Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of a Graphite/Epoxy Lamina.[13]



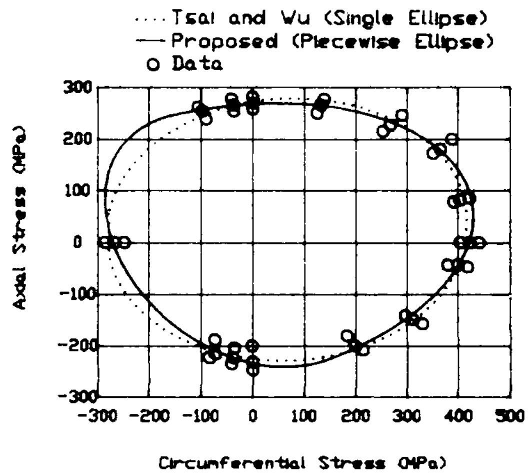
(Note 1 MPa = 0.145 Ksi)

Fig. 4 Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of a Graphite/Aluminum Lamina.[14]



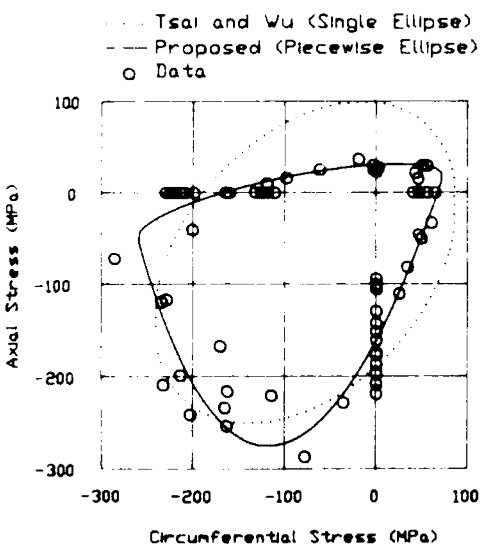
(Note 1 MPa = 0.145 Ksi)

Fig. 2 Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of a [0/90/0/90]$_s$ Graphite/Epoxy Laminate.[13]



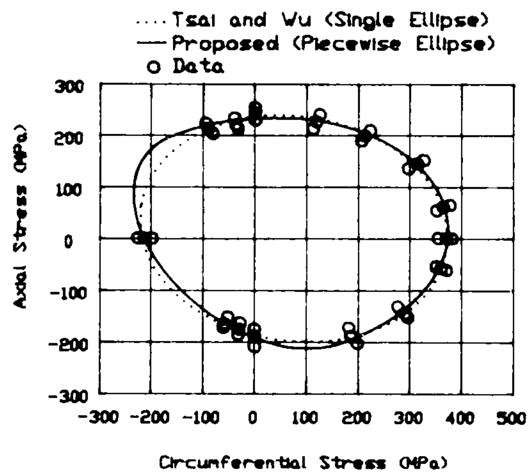
(Note 1 MPa = 0.145 Ksi)

Fig. 5 Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of a Satin-Weave Glass-Reinforced Plastic.[15]



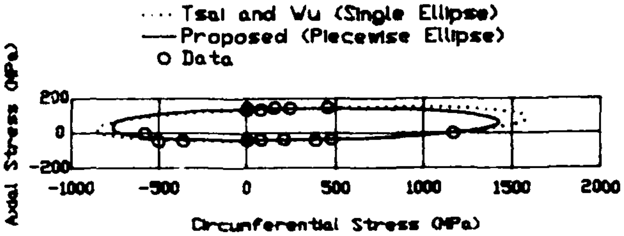
(Note 1 MPa = 0.145 Ksi)

Fig. 3 Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of 11-50 Composite Material.[10]



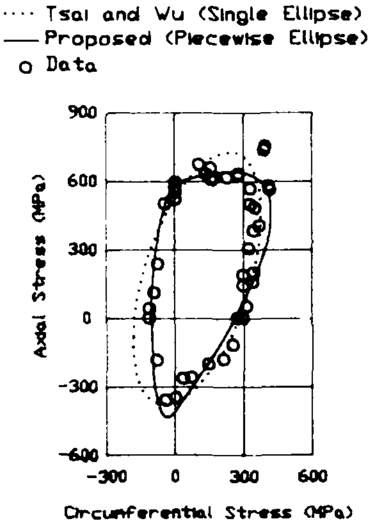
(Note 1 MPa = 0.145 Ksi)

Fig. 6 Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of a Linen-Weave Glass-Reinforced Plastic.[15]



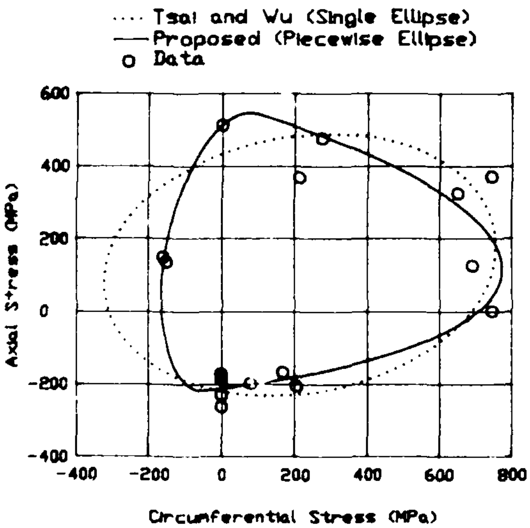
(Note: 1 MPa - 0.145 Ksi)

Fig. 7. Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of a Unidirectional Glass Epoxy Laminate.[16]



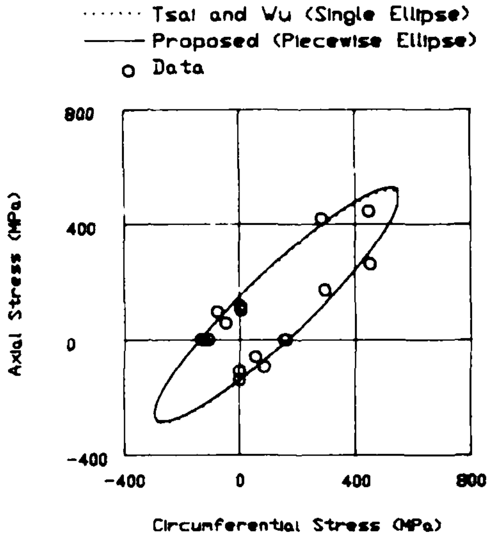
(Note: 1 MPa = 0.145 Ksi)

Fig. 8. Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of a [90 ±30 90] Glass Epoxy Laminate.[16]



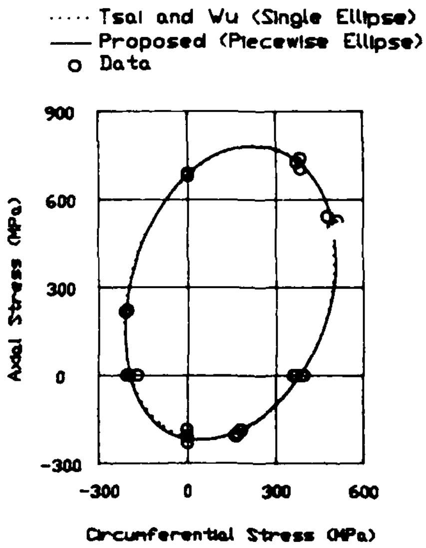
(Note: 1 MPa = 0.145 Ksi)

Fig. 9. Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of a Cross-Ply Glass Epoxy Laminate.[17]



(Note: 1 MPa = 0.145 Ksi)

Fig. 10. Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of a Helically Wound Glass Epoxy Laminate.[17]



(Note: 1 MPa = 0.145 Ksi)

Fig. 11. Correlations of the Quadratic Strength Tensor Theories with the Biaxial Fracture Data of an Organic Textolite.[18]